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On massive tensor multiplets

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Abstract: Massive tensor multiplets have recently been scrutinized in [1] and [2], as they appear in orientifold compactifications of type-IIB string theory. Here we formulate several dually equivalent models for massive $\mathcal{N} = 1, 2$ tensor multiplets in four space-time dimensions. In the $\mathcal{N} = 2$ case, we employ harmonic and projective superspace techniques.

Keywords: Supersymmetry and Duality, Extended Supersymmetry, Supergravity Models, Superspaces.

Contents

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1. Introduction

Recently, there has been renewed interest in 4D $\mathcal{N}=1$ massive tensor multiplets and their couplings to scalar and vector multiplets [1, 2]. Such interest is primarily motivated by the fact that massive two-forms naturally appear in four-dimensional $\mathcal{N}=2$ supergravity theories obtained from (or related to) compactifications of type-II string theory on Calabi-Yau threefolds in the presence of both electric and magnetic fluxes [3, 4]. This clearly provides enough ground for undertaking a more detailed study of massive $\mathcal{N}=1$ and $\mathcal{N}=2$ tensor multiplets.

In $\mathcal{N}=1$ supersymmetry, the massive tensor multiplet (as a dual version of the massive vector multiplet) was introduced twenty five years ago [5], and since then this construction[1] has been reviewed in two textbooks [8, 9]. In the original formulation [5], the mass parameter, m^2, in the action

$$S_{\text{tensor}} = -\frac{1}{2}\int \mathrm{d}^8 z\, G^2 - \frac{1}{2}\left\{m^2 \int \mathrm{d}^6 z\, \psi^\alpha \psi_\alpha + \text{c.c.}\right\} \tag{1.1}$$

was chosen to be real. Here

$$G = \frac{1}{2}(D^\alpha \psi_\alpha + \bar{D}_{\dot\alpha}\bar\psi^{\dot\alpha})\,, \qquad \bar{D}_{\dot\alpha}\psi_\alpha = 0\,, \tag{1.2}$$

where the dynamical variable ψ_α is an arbitrary chiral spinor superfield, and the (massless gauge) field strength G is a real linear superfield, $D^2 G = \bar{D}^2 G = 0$. The choice of a real mass parameter seemed to be natural in the sense that, for $M = m$, the above system is dual to the massive vector multiplet model[2]

$$S_{\text{vector}} = \frac{1}{4}\int \mathrm{d}^6 z\, W^\alpha W_\alpha + \frac{1}{2}M^2 \int \mathrm{d}^8 z\, V^2\,, \qquad W_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha V\,, \quad V = \bar{V}\,, \tag{1.3}$$

[1]The work of [5] is actually more famous for the massless tensor multiplet (as a dual version of the chiral scalar multiplet) introduced in it, see also [6, 7].

[2]This duality is a supersymmetric version of the duality between massive one- and two-forms [10].

which involves an intrinsically real mass parameter. The mass parameter is also intrinsically real in the vector-tensor realization [5] (inspired by [11])

$$\begin{aligned} S_{\mathrm{v-t}} &= -\frac{1}{2}\int \mathrm{d}^8z\, G^2 + \frac{1}{4}\int \mathrm{d}^6z\, W^\alpha W_\alpha + M\int \mathrm{d}^8z\, G\, V \\ &= -\frac{1}{2}\int \mathrm{d}^8z\, G^2 + \frac{1}{4}\int \mathrm{d}^6z\, W^\alpha W_\alpha - \frac{1}{2}M\left\{\int \mathrm{d}^6z\, W^\alpha\psi_\alpha + \mathrm{c.c.}\right\}, \end{aligned} \tag{1.4}$$

which describes the same multiplet on the mass shell (massive superspin-1/2), and which possesses both the tensor multiplet gauge freedom

$$\delta\psi_\alpha = \frac{\mathrm{i}}{4}\bar{D}^2 D_\alpha K\,, \qquad K = \bar{K} \tag{1.5}$$

and the vector multiplet gauge freedom

$$\delta V = \Lambda + \bar{\Lambda}\,, \qquad \bar{D}_{\dot\alpha}\Lambda = 0\,. \tag{1.6}$$

It was recently pointed out [1, 2], however, that giving the mass parameter in (1.1) an imaginary part,[3]

$$m^2 \quad\longrightarrow\quad m(m+\mathrm{i}e)\,, \tag{1.7}$$

leads to nontrivial physical implications, for the mass can now be interpreted to have both electric and magnetic contributions which are associated with the two possible mass terms $B \wedge {}^*B$ and $B \wedge B$ for the component two-form. The dual vector multiplet is then characterized by the mass $M = \sqrt{m^2 + e^2}$. What makes the replacement (1.7) really interesting is that the complex mass parameter can be interpreted as a vacuum expectation value for chiral scalars [12],

$$m(m+\mathrm{i}e)\int \mathrm{d}^6z\, \psi^\alpha\psi_\alpha \quad\longleftarrow\quad \int \mathrm{d}^6z\, F(\phi)\, \psi^\alpha\psi_\alpha\,, \tag{1.8}$$

with ϕ some chiral scalars, $\bar{D}_{\dot\alpha}\phi = 0$.

In the massive case, the gauge freedom (1.5) is broken if one does not use the vector-tensor formulation (1.4). It can be restored, however, if one implements the standard Stueckelberg formalism, as was done in [1, 2], and replaces the naked chiral prepotential ψ_α everywhere by

$$\psi_\alpha \quad\longrightarrow\quad \psi_\alpha + \frac{\mathrm{i}}{m}W_\alpha\,, \tag{1.9}$$

where the compensating vector multiplet has to transform as $\delta V = mK$ under (1.5). The gauge symmetry thus obtained can be treated as a deformation of the transformations (1.5) and (1.6).

In this note we continue the research started in [1, 2] and provide further insight into the structure of massive tensor multiplets. In section 2 we consider aspects of $\mathcal{N} = 1$ tensor multiplets in curved superspace and introduce a model for the massive improved

[3] Unlike the scalar multiplet, this imaginary part cannot be eliminated by a rigid phase transformation of ψ_α as long as the explicit form of the linear scalar G, in terms of ψ_α and its conjugate, is fixed.

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tensor multiplet. Unlike the ordinary tensor multiplet [5], the (massless) improved tensor multipet [13] is superconformal in global supersymmetry and super Weyl invariant in curved superspace. There are at least two reasons why the improved tensor multiplet is interesting: (i) it describes the superconformal compensator in the new minimal formulation of $\mathcal{N}=1$ supergravity, see [8, 9] for reviews; (ii) it corresponds to the Goldstone multiplet for partial breaking of $\mathcal{N}=1$ superconformal symmetry associated with the coset $\mathrm{SU}(2,2|1)/(\mathrm{SO}(4,1)\times\mathrm{U}(1))$ which has AdS_5 as the bosonic subspace [14, 15]. As we demonstrate below, a remarkable feature of the improved tensor multiplet is that its super Weyl invariance remains intact in the massive case.

In section 3 we introduce several realizations for the massive $\mathcal{N}=2$ tensor multiplet, describe its duality to the massive $\mathcal{N}=2$ vector multiplet, and also sketch possible self-couplings and couplings to vector multiplets. Section 4 is devoted to the description of the reduction of manifestly $\mathcal{N}=2$ supersymmetric actions to $\mathcal{N}=1$ superspace. The list of $\mathcal{N}=2$ superspace integrations measures is given in the appendix.

Our $\mathcal{N}=1$ supergravity conventions correspond to [9]. They are very similar to those adopted in [16]. The conversion from [9] to [16] is as follows: $E^{-1}\to E$ and $R\to 2R$.

2. $\mathcal{N}=1$ tensor multiplets

It is known that 4D $\mathcal{N}=1$ new minimal supergravity can be treated as a super Weyl invariant dynamical system describing the coupling of old minimal supergravity to a *real* covariantly linear scalar superfield $\mathbb{L}$ constrained by $(\bar{\mathcal{D}}^2-4R)\,\mathbb{L}=(\mathcal{D}^2-4\bar{R})\,\mathbb{L}=0$, see [8, 9] for reviews.[4] The new minimal supergravity action

$$S_{\mathrm{SG,new}}=\frac{3}{\kappa^2}\int \mathrm{d}^8z\,E^{-1}\,\mathbb{L}\ln\mathbb{L}\,,\qquad E=\mathrm{Ber}(E_A{}^M) \tag{2.1}$$

is invariant with respect to the super Weyl transformation[5] [17]

$$\mathcal{D}_\alpha\;\to\;\mathrm{e}^{\sigma/2-\bar\sigma}\Big(\mathcal{D}_\alpha-(\mathcal{D}^\beta\sigma)\,M_{\alpha\beta}\Big)\,,\qquad \bar{\mathcal{D}}_{\dot\alpha}\;\to\;\mathrm{e}^{\bar\sigma/2-\sigma}\Big(\bar{\mathcal{D}}_{\dot\alpha}-(\bar{\mathcal{D}}^{\dot\beta}\bar\sigma)\bar{M}_{\dot\beta\dot\alpha}\Big) \tag{2.2}$$

accompanied with

$$\mathbb{L}\;\to\;\mathrm{e}^{-\sigma-\bar\sigma}\,\mathbb{L}. \tag{2.3}$$

where $\sigma(z)$ is an arbitrary covariantly chiral scalar parameter, $\bar{\mathcal{D}}_{\dot\alpha}\sigma=0$. The super Weyl transformation of $\mathbb{L}$ is uniquely fixed by the constraint (2.6), with G replaced by $\mathbb{L}$. The dynamical system (2.1) is classically equivalent to old minimal supergravity described by the action

$$S_{\mathrm{SG,old}}=-\frac{3}{\kappa^2}\int \mathrm{d}^8z\,E^{-1}\,. \tag{2.4}$$

[4] In old minimal supergravity, the superspace covariant derivatives are $\mathcal{D}_A=(\mathcal{D}_a,\mathcal{D}_\alpha,\bar{\mathcal{D}}^{\dot\alpha})=E_A{}^M(z)\,\partial_M+\Omega_A{}^{\beta\gamma}(z)\,M_{\beta\gamma}+\Omega_A{}^{\dot\beta\dot\gamma}(z)\,\bar{M}_{\dot\beta\dot\gamma}$, with $M_{\beta\gamma}$ and $\bar{M}_{\dot\beta\dot\gamma}$ the Lorentz generators. They obey the (modified) Wess-Zumino constraints, and the latter imply that the torsion and the curvature are expressed in terms of a vector $G_a=\bar{G}_a$ and covariantly chiral objects R and $W_{\alpha\beta\gamma}$ subject to some additional Bianchi identities.

[5] Under (2.2), the full superspace measure changes as $\mathrm{d}^8z\,E^{-1}\to\mathrm{d}^8z\,E^{-1}\,\exp(\sigma+\bar\sigma)$, while the *chiral superspace measure* transforms as $\mathrm{d}^8z\,E^{-1}/R\to\mathrm{d}^8z\,(E^{-1}/R)\,\exp(3\sigma)$.

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Modulo sign, the functional (2.1) coincides with the action for the so-called improved tensor multiplet [13]

$$S = -\mu \int \mathrm{d}^8 z \, E^{-1} \, G \ln(G/\mu) \,, \tag{2.5}$$

with G obeying the same constraint as $\mathbb{L}$ above,

$$(\bar{\mathcal{D}}^2 - 4R) \, G = (\mathcal{D}^2 - 4\bar{R}) \, G = 0 \,. \tag{2.6}$$

In the family of tensor multiplet models [5] of the form

$$S = \mu^2 \int \mathrm{d}^8 z \, E^{-1} \, \mathcal{F}\Big(\frac{G}{\mu}\Big) \,, \tag{2.7}$$

the action (2.5) is singled out by the requirement of super Weyl invariance. In particular, the free massless tensor multiplet action

$$S = -\frac{1}{2} \int \mathrm{d}^8 z \, E^{-1} \, G^2 \tag{2.8}$$

is not super Weyl invariant. Upon reduction to flat superspace, the action (2.5) becomes superconformal.

As is well known, the general solution of (2.6) is

$$G = \frac{1}{2}(\mathcal{D}^\alpha \psi_\alpha + \bar{\mathcal{D}}_{\dot\alpha} \bar\psi^{\dot\alpha}) \,, \qquad \bar{\mathcal{D}}_{\dot\alpha} \psi_\alpha = 0 \,, \tag{2.9}$$

with an arbitrary covariantly chiral spinor superfield ψ_α. The super Weyl transformation of the prepotential ψ_α turns out to be uniquely fixed [9]:

$$G \; \to \; \mathrm{e}^{-\sigma - \bar\sigma} \, G \quad \Longrightarrow \quad \psi_\alpha \; \to \; \mathrm{e}^{-3\sigma/2} \, \psi_\alpha \,. \tag{2.10}$$

As a result, adding a mass term to the action (2.5) does not spoil its super Weyl invariance. That is, the action

$$S[\psi, \bar\psi] = -\mu \int \mathrm{d}^8 z \, E^{-1} \, G \ln\Big(\frac{G}{\mu}\Big) - \frac{1}{2} m \Big\{ (m + \mathrm{i}\, e) \int \mathrm{d}^8 z \, \frac{E^{-1}}{R} \, \psi^2 + \mathrm{c.c.} \Big\} \tag{2.11}$$

is invariant under arbitrary super Weyl transformations. The latter property uniquely singles out this model in the family of actions

$$S = \mu^2 \int \mathrm{d}^8 z \, E^{-1} \, \mathcal{F}\Big(\frac{G}{\mu}\Big) - \frac{1}{2} m \Big\{ (m + \mathrm{i}\, e) \int \mathrm{d}^8 z \, \frac{E^{-1}}{R} \, \psi^2 + \mathrm{c.c.} \Big\} \,. \tag{2.12}$$

Therefore, the action (2.11) defines the massive improved tensor multiplet. This is a nontrivial theory, unlike the massless improved tensor multiplet that is known to be free. Upon reduction to flat superspace, the action turns into a superconformal model.

Let us consider a dual formulation for the theory introduced in (2.11). We follow the procedure given in [5, 8, 9] and first relax the linear constraints $(\bar{\mathcal{D}}^2 - 4R) \, G = (\mathcal{D}^2 - 4\bar{R}) \, G = 0$ by introducing the "first-order" model

$$\begin{aligned} S_{\text{auxiliary}} = & -\mu \int \mathrm{d}^8 z \, E^{-1} \, G \Big(\ln\Big(\frac{G}{\mu}\Big) - 1 \Big) + M \int \mathrm{d}^8 z \, E^{-1} \, V \Big(G - \frac{1}{2}(\mathcal{D}^\alpha \psi_\alpha + \bar{\mathcal{D}}_{\dot\alpha} \bar\psi^{\dot\alpha}) \Big) - \\ & - \frac{1}{2} m \Big\{ (m + \mathrm{i}\, e) \int \mathrm{d}^8 z \, \frac{E^{-1}}{R} \, \psi^2 + \mathrm{c.c.} \Big\} \,, \end{aligned} \tag{2.13}$$

with

$$M^2 = m^2 + e^2 \,. \tag{2.14}$$

Here both scalars G and V are real unconstrained, and G is not related to ψ_α and its conjugate. To preserve the super Weyl invariance, however, V should transform as follows:

$$V \;\to\; V - \frac{\mu}{M}\,(\sigma + \bar\sigma)\,. \tag{2.15}$$

Varying $S_{\rm auxiliary}$ with respect to V brings us back to (2.11). On the other hand, varying $S_{\rm auxiliary}$ with respect to G and ψ_α allows one to express these variables in terms of V and the vector multiplet strength

$$W_\alpha = -\frac{1}{4}(\bar{\mathcal D}^2 - 4R)\mathcal{D}_\alpha V\,, \qquad \bar{\mathcal D}_{\dot\alpha} W_\alpha = 0\,, \qquad \mathcal{D}^\alpha W_\alpha = \bar{\mathcal D}_{\dot\alpha} \bar W^{\dot\alpha}\,. \tag{2.16}$$

One ends up with

$$S[V] = \frac{1}{4}\int {\rm d}^8 z\,\frac{E^{-1}}{R}\,W^2 + \mu^2 \int {\rm d}^8 z\, E^{-1}\,\exp\left(\frac{M}{\mu}V\right). \tag{2.17}$$

This action is invariant under the super Weyl transformations (2.2) and (2.15). It is worth pointing out that the inhomogeneous piece on the right of (2.15) does not show up in the transformation of W_α:

$$W_\alpha \;\to\; {\rm e}^{-3\sigma/2}\,W_\alpha\,. \tag{2.18}$$

The super Weyl transformations of the chiral spinors ψ_α and W_α are clearly identical.

Employing the Stueckelberg formalism, the action (2.17) can be replaced by the classically-equivalent one

$$S[V,\Phi,\bar\Phi] = \frac{1}{4}\int {\rm d}^8 z\,\frac{E^{-1}}{R}\,W^2 + \mu^2 \int {\rm d}^8 z\, E^{-1}\,\bar\Phi\,{\rm e}^{(M/\mu)V}\,\Phi\,, \tag{2.19}$$

with Φ a compensating chiral scalar possessing a non-vanishing v.e.v. This action is invariant under the U(1) gauge transformation

$$\delta V = \Lambda + \bar\Lambda\,, \qquad \delta\Phi = -\frac{\mu}{M}\,\Lambda\,\Phi\,, \qquad \bar{\mathcal D}_{\dot\alpha}\Lambda = 0\,. \tag{2.20}$$

The super Weyl transformation (2.15) turns into

$$V \;\to\; V\,, \qquad \Phi \;\to\; {\rm e}^{-\sigma}\,\Phi\,. \tag{2.21}$$

The model (2.17), or its equivalent realization (2.19), describes the dynamics of a massive improved vector multiplet in curved superspace.

The mass term in (2.11) breaks the massless gauge symmetry [5]

$$\delta\psi_\alpha = \frac{\rm i}{4}(\bar{\mathcal D}^2 - 4R)\mathcal{D}_\alpha K\,, \qquad K = \bar K \tag{2.22}$$

that leaves the field strength (2.9) invariant. Nevertheless, inspired by [11], one can preserve the gauge symmetry in the massive case by considering the following vector-tensor model

$$S[\psi,\bar\psi,V] = -\mu\int {\rm d}^8 z\, E^{-1}\, G\ln\left(\frac{G}{\mu}\right) + \frac{1}{4}\int {\rm d}^8 z\,\frac{E^{-1}}{R}\,W^2 + M\int {\rm d}^8 z\, E^{-1}\, G\,V\,. \tag{2.23}$$

This action possesses both the tensor multiplet (2.22) and vector multiplet (2.20) gauge symmetries. This action can also be seen to be super Weyl invariant provided V is chosen, say, to be inert under such transformations. By inspecting the equations of motion, one can check that the theory (2.23) is classically equivalent to (2.11) if M is chosen as in (2.14). One can also establish the duality of (2.23) to the improved vector multiplet (2.19) by dualizing the linear superfield G into a chiral scalar and its conjugate according to [5, 7].

In the massive case, following Stueckelberg, the gauge invariance (2.22) can be restored by introducing a compensating abelian vector multiplet (with the gauge field $\mathcal{V}$ and the chiral field strength $\mathcal{W}_\alpha$) and replacing ψ_α in (2.12) by

$$\psi_\alpha \quad \longrightarrow \quad \psi_\alpha + \frac{\mathrm{i}}{m}\mathcal{W}_\alpha\,, \qquad \mathcal{W}_\alpha = -\frac{1}{4}(\bar{\mathcal{D}}^2 - 4R)\mathcal{D}_\alpha \mathcal{V}\,, \quad \mathcal{V} = \bar{\mathcal{V}}\,. \tag{2.24}$$

Here $\mathcal{V}$ transforms as $\delta\mathcal{V} = m\,K$ under (2.22) such that the combination $m\,\psi_\alpha + \mathrm{i}\,\mathcal{W}_\alpha$ is gauge invariant. The modified mass term remains to be super Weyl invariant. Since

$$\mathrm{Im}\int \mathrm{d}^8 z\, \frac{E^{-1}}{R}\mathcal{W}^2 = 0\,,$$

we then obtain

$$\mathrm{i}\,m\int \mathrm{d}^8 z\, \frac{E^{-1}}{R}\Big(\psi + \frac{\mathrm{i}}{m}\mathcal{W}\Big)^2 + \text{c.c.} = \mathrm{i}\int \mathrm{d}^8 z\, \frac{E^{-1}}{R}\Big(m\psi^2 + 2\mathrm{i}\,\psi\mathcal{W}\Big) + \text{c.c.} \tag{2.25}$$

3. $\mathcal{N} = 2$ tensor multiplets

To generalize the previous consideration to the case of $\mathcal{N} = 2$ supersymmetry, it is advantageous (in some respect, necessary) to make use of the $\mathcal{N} = 2$ harmonic superspace $\mathbb{R}^{4|8} \times S^2$ [18, 19]. It extends conventional $\mathcal{N} = 2$ superspace $\mathbb{R}^{4|8}$ (paramerized by coordinates $Z = (x^a, \theta^\alpha_i, \bar\theta^i_{\dot\alpha})$, with $i = \hat 1, \hat 2$) by the two-sphere $S^2 = \mathrm{SU}(2)/\,\mathrm{U}(1)$ parametrized by harmonics, i.e., group elements

$$(u_i{}^-\,, u_i{}^+) \in \mathrm{SU}(2)\,, \quad u^+_i = \varepsilon_{ij}u^{+j}\,, \quad \overline{u^{+i}} = u^-_i\,, \quad u^{+i}u^-_i = 1\,. \tag{3.1}$$

For simplicity, our consideration will be restricted to the study of globally supersymmetric theories only.

Let us start by recalling the model for a free massive $\mathcal{N} = 2$ vector multiplet [18, 19]. Its dynamical variable $V^{++}(Z, u)$ is a real analytic superfield, $D^+_\alpha V^{++} = \bar D^+_{\dot\alpha} V^{++} = 0$, where the harmonic-dependent spinor covariant derivatives $D^\pm_\alpha$ and $\bar D^\pm_{\dot\alpha}$ are defined in eq. (A.5). The action[6] is

$$\begin{aligned} S_{\rm vector} &= \frac{1}{2}\int \mathrm{d}^8 Z\, W^2 - \frac{1}{2}M^2\int \mathrm{d}\zeta^{(-4)}\,(V^{++})^2 \\ &= \frac{1}{2}\int \mathrm{d}^{12}Z\,\mathrm{d}u\mathrm{d}u'\,\frac{V^{++}(u)\,V^{++}(u')}{(u^+u'^+)^2} - \frac{1}{2}M^2\int \mathrm{d}\zeta^{(-4)}\,(V^{++})^2\,, \end{aligned} \tag{3.2}$$

[6]The various $\mathcal{N} = 2$ superspace integration measures are defined in the appendix.

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see [19] for the definition of harmonic distributions of the form $(u_1^+ u_2^+)^{-n}$, where $(u_1^+ u_2^+) = u_1^{+i} u_{2\,i}^+$. Here $W(Z)$ is the (harmonic independent) chiral field strength of the $\mathcal{N} = 2$ vector multiplet [20],

$$\bar{D}^i_{\dot\alpha} W = 0\,, \qquad D^{\alpha i} D^j_\alpha W = \bar{D}^i_{\dot\alpha} \bar{D}^{j\dot\alpha} \bar{W}\,, \tag{3.3}$$

which is expressed via the analytic prepotential $V^{++}(Z,u)$ as follows [19, 21]:

$$\begin{aligned} W(Z) = \frac{1}{4} \int \mathrm{d}u\, (\bar{D}^-)^2\, V^{++}(Z,u) \;&=\; \frac{1}{4} (\bar{D}^+)^2\, V^{--}(Z,u)\,, \\ V^{--}(Z,u) \;&=\; \int \mathrm{d}u'\, \frac{V^{++}(Z,u')}{(u^+ u'^+)^2}\,. \end{aligned} \tag{3.4}$$

The equation of motion is

$$\frac{1}{4} (D^+)^2 W - M^2\, V^{++} = 0\,, \tag{3.5}$$

where one should keep in mind that the Bianchi identity is equivalent to $(D^+)^2\, W = (\bar{D}^+)^2 \bar{W}$. This equation implies that V^{++} is an $\mathcal{N} = 2$ linear superfield:

$$D^{++} V^{++} = 0 \quad \longrightarrow \quad V^{++}(Z,u) = V^{(ij)}(Z) u_i^+ u_j^+\,, \tag{3.6}$$

where V^{ij} obeys the constaints

$$D^{(i}_\alpha V^{jk)} = \bar{D}^{(i}_{\dot\alpha} V^{jk)} = 0 \quad \longleftarrow \quad D^+_\alpha V^{++} = \bar{D}^+_{\dot\alpha} V^{++} = 0\,, \tag{3.7}$$

as a consequence of the analyticity of the dynamical variable. It is now easy to arrive at

$$(\Box - M^2) V^{++} = 0 \quad \longrightarrow \quad (\Box - M^2) W = 0\,. \tag{3.8}$$

In the massless case, $M = 0$, the action (3.2) is invariant under the gauge transformation [18, 19]

$$\delta V^{++} = D^{++} \lambda\,, \tag{3.9}$$

with the gauge parameter $\lambda(Z,u)$ a real analytic superfield, $D^+_\alpha \lambda = \bar{D}^+_{\dot\alpha} \lambda = 0$. This transformation leaves the field strength (3.4) invariant.

Let us now turn to the massless $\mathcal{N} = 2$ tensor multiplet [22] formulated in harmonic superspace in [23, 19]. The free action is

$$S = \frac{1}{2} \int \mathrm{d}\zeta^{(-4)}\, (G^{++})^2\,, \tag{3.10}$$

where $G^{++}(Z,u)$ is a restricted real analytic superfield under the constraints (3.6) and (3.7). One can express $G^{++}(Z,u) = G^{ij}(Z)\, u_i^+ u_j^+$ in terms of an unconstrained chiral superfield $\Psi(Z)$ and its conjugate:

$$G^{++}(Z,u) = \frac{1}{8} (D^+)^2\, \Psi(Z) + \frac{1}{8} (\bar{D}^+)^2\, \bar{\Psi}(Z)\,, \qquad \bar{D}^i_{\dot\alpha} \Psi = 0\,. \tag{3.11}$$

This superfield remains invariant under the gauge transformation

$$\delta\Psi = \mathrm{i}\, \Lambda\,, \qquad \bar{D}^i_{\dot\alpha} \Lambda = 0\,, \qquad D^{\alpha i} D^j_\alpha \Lambda = \bar{D}^i_{\dot\alpha} \bar{D}^{j\dot\alpha} \bar{\Lambda}\,. \tag{3.12}$$

As is seen, the chiral gauge parameter Λ satisfies the same constraints as the vector multiplet field strength.

Recalling the construction of [11], the massive tensor (or vector) multiplet can be described by the action

$$
\begin{aligned}
S_{\rm v-t} &= \frac{1}{2}\int {\rm d}\zeta^{(-4)}\,(G^{++})^2 + \frac{1}{2}\int {\rm d}^8Z\,W^2 + M\int {\rm d}\zeta^{(-4)}\,G^{++}\,V^{++} \qquad (3.13)\\
&= \frac{1}{2}\int {\rm d}\zeta^{(-4)}\,(G^{++})^2 + \frac{1}{2}\int {\rm d}^8Z\,W^2 + \frac{1}{2}M\Big\{\int {\rm d}^8Z\,W\Psi + {\rm c.c.}\Big\}\,, \qquad (3.14)
\end{aligned}
$$

which is invariant under the gauge transformations (3.9) and (3.12). The corresponding equations of motion are

$$
\frac{1}{4}(\bar{D}^-)^2G^{++} + M\,W = 0\,, \qquad \frac{1}{4}(D^+)^2W + MG^{++} = 0\,, \qquad (3.15)
$$

as well as the complex conjugate of the first equation.

Of primary importance for us will be another massive extension of (3.10)

$$
S_{\rm tensor} = \frac{1}{2}\int {\rm d}\zeta^{(-4)}\,(G^{++})^2 - \frac{1}{4}m\Big\{(m+{\rm ie})\int {\rm d}^8Z\,\Psi^2 + {\rm c.c.}\Big\}\,. \qquad (3.16)
$$

This action generates the following equations of motion

$$
\frac{1}{4}(\bar{D}^-)^2G^{++} - m(m+{\rm ie})\,\Psi = 0\,, \qquad \frac{1}{4}(D^-)^2G^{++} - m(m-{\rm ie})\,\bar{\Psi} = 0\,, \qquad (3.17)
$$

which imply

$$
(\Box - M^2)G^{++} = 0\,, \qquad M = \sqrt{m^2+e^2}\,. \qquad (3.18)
$$

The dynamical systems (3.2) and (3.16) turn out to be dual to each other provided M is chosen as above. The corresponding "first-order" action, which establishes the duality between these theories, is

$$
\begin{aligned}
S_{\rm auxiliary} &= \frac{1}{2}\int {\rm d}\zeta^{(-4)}\,(G^{++})^2 + \frac{1}{8}M\int {\rm d}\zeta^{(-4)}\,V^{++}\Big(8G^{++} - (D^+)^2\,\Psi - (\bar{D}^+)^2\,\bar{\Psi}\Big)\\
&\quad - \frac{1}{4}m\Big\{(m+{\rm ie})\int {\rm d}^8Z\,\Psi^2 + {\rm c.c.}\Big\}\,, \qquad (3.19)
\end{aligned}
$$

where both real analytic superfields V^{++} and G^{++} are unconstrained. Varying V^{++} brings us back to (3.16). On the other hand, varying $S_{\rm auxiliary}$ with respect to G^{++} and Ψ and using the equations obtained to eliminate these superfields, we end up with (3.2).

One can also establish duality between (3.2) and the gauge-invariant model (3.13) by using the known duality between the massless tensor multipet and the ω-hypermultiplet [23]. Consider the "first-order" action

$$
\tilde{S}_{\rm v-t} = \int {\rm d}\zeta^{(-4)}\,\Big\{\frac{1}{2}(G^{++})^2 + MG^{++}\,V^{++} + G^{++}\,D^{++}\omega\Big\} + \frac{1}{2}\int {\rm d}^8Z\,W^2\,, \qquad (3.20)
$$

in which G^{++} and ω are real unrestricted analytic superfields. Varying ω gives $D^{++}G^{++} = 0$, and then (3.13) is reproduced. On the other hand, varying G^{++} and then eliminating it from $\tilde{S}_{\rm v-t}$, we arrive at the action

$$\hat{S}_{\rm vector} = \frac{1}{2}\int {\rm d}^8 Z\, W^2 - \frac{1}{2}M^2 \int {\rm d}\zeta^{(-4)} \left(V^{++} + \frac{1}{M}\, D^{++}\omega\right)^2 , \qquad (3.21)$$

and this is simply the Stueckelberg*ization* of (3.2).

In the massive case, the gauge freedom (3.12) is broken. It can be restored, following Stueckelberg, by introducing a compensating abelian vector multiplet (with the gauge field $\mathcal{V}^{++}$ and the chiral field strength $\mathcal{W}$) and replacing the naked prepotential Ψ as follows:

$$\Psi \quad \longrightarrow \quad \Psi + \frac{\rm i}{m}\mathcal{W}\,, \qquad \delta\mathcal{W} = -m\Lambda\,. \qquad (3.22)$$

The combination $m\Psi + {\rm i}\,\mathcal{W}$ is invariant under the gauge transformations (3.12). One then obtains

$$m\,{\rm i}\int {\rm d}^8 Z \left(\Psi + \frac{\rm i}{m}\mathcal{W}\right)^2 + {\rm c.c.} = {\rm i}\int {\rm d}^8 Z \left(m\,\Psi^2 + 2{\rm i}\,\Psi\,\mathcal{W}\right) + {\rm c.c.} \qquad (3.23)$$

Up to this point, the use of $\mathcal{N}=2$ harmonic superspace has allowed us to keep a complete analogy with the $\mathcal{N}=1$ case previously considered.

Let us turn to possible generalizations to generate (self-)interactions. A natural extension[7] of the kinetic term (3.10) is [23, 19]

$$\frac{1}{2}\int {\rm d}\zeta^{(-4)}\,(G^{++})^2 \quad \longrightarrow \quad S_{\rm H} = \int {\rm d}\zeta^{(-4)}\,\mathcal{L}^{(+4)}(G^{++}, u^+, u^-)\,. \qquad (3.24)$$

Here $\mathcal{L}^{(+4)}$ is an arbitrary (real analytic) function of the field strength G^{++} and the harmonics $u^\pm$ carrying U(1) charge +4. In particular, the improved $\mathcal{N}=2$ tensor multiplet [24, 7, 25, 23] is generated by [23, 19]

$$\mathcal{L}^{(+4)}_{\rm impr}(G^{++}, u) = \mu^2 \left(\frac{\mathcal{G}^{++}}{1 + \sqrt{1 + \mathcal{G}^{++}\,c^{--}}}\right)^2 , \qquad \mathcal{G}^{++} = \frac{G^{++}}{\mu} - c^{++}\,, \qquad (3.25)$$

with c^{++} a holomorphic vector field on S^2,

$$c^{\pm\pm}(u) = c^{ij}\,u_i^\pm u_j^\pm\,, \qquad c^{ij}c_{ij} = 2\,, \qquad c^{ij} = {\rm const}\,. \qquad (3.26)$$

The corresponding action takes a simpler form in the so-called projective superspace [25, 26].

We can introduce the massive improved tensor multiplet

$$S_{\rm impr} = \int {\rm d}\zeta^{(-4)}\,\mathcal{L}^{(+4)}_{\rm impr}(G^{++}, u) - \frac{1}{4}m\Big\{(m + {\rm i}e)\int {\rm d}^8 Z\,\Psi^2 + {\rm c.c.}\Big\}\,. \qquad (3.27)$$

[7]One can also add two supersymmetric gauge-invariant terms ${\rm Re}\Big\{c_1\int {\rm d}\zeta^{(-4)}\,G^{++}(\theta^+)^2 + c_2\int {\rm d}^8 Z\,\Psi\Big\}$, with $c_{1,2}$ complex parameters, which trigger spontaneous supersymmetry breaking.

Here the kinetic term is known to be invariant under the $\mathcal{N}=2$ superconformal group [23, 19]. The mass term turns out to be superconformally invariant as well. In fact, the above action is the most general superconformal action without higher derivatives. Instead of (3.27), we can work with the gauge-invariant action

$$S'_{\rm impr} = \int {\rm d}\zeta^{(-4)}\, \mathcal{L}^{(+4)}_{\rm impr}(G^{++},u) + \frac{1}{2}\int {\rm d}^8 Z\, W^2 + M\int {\rm d}\zeta^{(-4)}\, G^{++}\, V^{++}\,, \qquad (3.28)$$

which respects the gauge symmetries (3.9) and (3.12). The linear superfield G^{++} in (3.28) can be further dualized into a real analytic superfield ω (ω-hypermultiplet), thus converting the action (3.28) into that for a massive improved vector multiplet.

The mass term in (3.16) admits a natural generalization of the form

$$m(m+{\rm i}e)\int {\rm d}^8 Z\,\Psi^2 \quad\longrightarrow\quad \int {\rm d}^8 Z\,\Upsilon(\Psi,\mathbb{W})\,, \qquad (3.29)$$

where Υ is a holomorphic function, and $\mathbb{W}$ stands for the chiral field strength(s) of some vector multiplet(s). Unlike the $\mathcal{N}=1$ supersymmetric case, where the chiral prepotential ψ_α was anticommuting, here we have no inherent reasons to insist that $\Upsilon(\Psi,\mathbb{W})$ be quadratic in Ψ.

4. From $\mathcal{N}=2$ superfields to $\mathcal{N}=1$ superfields

Here we describe the reduction of $\mathcal{N}=2$ tensor multiplet models to $\mathcal{N}=1$ superspace. The latter is parametrized by the coordinates $z=(x^a,\theta^\alpha,\bar\theta_{\dot\alpha})$ related to the $\mathcal{N}=2$ superspace coordinates $Z=(x^a,\theta^\alpha_i,\bar\theta^i_{\dot\alpha})$, with $i=\hat1,\hat2$, as follows:

$$\theta^\alpha = \theta_{\hat1}{}^\alpha\,, \qquad \bar\theta_{\dot\alpha} = \bar\theta^{\hat1}{}_{\dot\alpha}\,. \qquad (4.1)$$

The $\mathcal{N}=1$ spinor covariant derivatives $(D_\alpha,\bar D^{\dot\alpha})$ are related to the $\mathcal{N}=2$ covariant derivatives $(D^i_\alpha,\bar D^{\dot\alpha}_i)$ in a similar fashion,

$$D_\alpha = D^{\hat1}{}_\alpha\,, \qquad \bar D^{\dot\alpha} = \bar D_{\hat1}{}^{\dot\alpha}\,. \qquad (4.2)$$

For any $\mathcal{N}=2$ superfield $U(Z)=U(x,\theta_i,\bar\theta^j)$, we define its $\mathcal{N}=1$ projection

$$U| = U(z) = U(x,\theta_i,\bar\theta^j)\Big|_{\theta_{\hat2}=\bar\theta^{\hat2}=0}\,. \qquad (4.3)$$

Start with the $\mathcal{N}=2$ tensor multiplet strength

$$G^{ij} = G^{ji} = \frac{1}{8}D^iD^j\,\Psi + \frac{1}{8}\bar D^i\bar D^j\,\bar\Psi\,, \qquad \bar D^i_{\dot\alpha}\Psi = 0\,, \qquad (4.4)$$

which obeys the constraints

$$D^{(i}_\alpha G^{jk)} = \bar D^{(i}_{\dot\alpha} G^{jk)} = 0\,. \qquad (4.5)$$

The prepotential $\Psi(Z)$ reduces to the three $\mathcal{N}=1$ chiral components:

$$\sigma = \Psi|\,, \qquad {\rm i}\,\psi_\alpha = \frac{1}{2}D^{\hat2}{}_\alpha\,\Psi|\,, \qquad \rho = -\frac{1}{4}D^{\hat2}D^{\hat2}\,\Psi|\,. \qquad (4.6)$$

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Then, for the components of G^{ij} we get

$$G^{\hat{2}\hat{2}}| \equiv \Phi = -\frac{1}{2}\left(\rho - \frac{1}{4}\bar{D}^2\bar{\sigma}\right), \qquad G^{\hat{2}\hat{1}}| = \frac{\mathrm{i}}{2}\,G\,, \qquad G^{\hat{1}\hat{1}}| = \bar{\Phi}\,, \tag{4.7}$$

with G the $\mathcal{N}=1$ tensor multiplet strength, eq. (1.2).

Now, for the $\mathcal{N}=2$ chiral mass-like term (3.29) with $\mathbb{W}=0$ we obtain

$$\int \mathrm{d}^8 Z\,\Upsilon(\Psi) = -\int \mathrm{d}^8 z\,\bar{\sigma}^I\,\Upsilon_I(\sigma) + \int \mathrm{d}^6 z\,\Big\{\psi^I\psi^J\,\Upsilon_{IJ}(\sigma) - 2\Phi^I\,\Upsilon_I(\sigma)\Big\}\,, \tag{4.8}$$

where we have specialized to the case of several tensor multiplets. This is, of course, very similar to the $\mathcal{N}=1$ form of the holomorphic prepotential in the Seiberg-Witten theory.

To reduce the free kinetic term for the tensor multiplet, eq. (3.10), to $\mathcal{N}=1$ superspace, one can use the identity [27]

$$\int \mathrm{d}\zeta^{(-4)}\,F^{++}\,G^{++} = \int \mathrm{d}^8 z\,\Big\{F^{\hat{1}\hat{1}}|\,G^{\hat{2}\hat{2}}| + F^{\hat{2}\hat{2}}|\,G^{\hat{1}\hat{1}}| + 4F^{\hat{1}\hat{2}}|\,G^{\hat{1}\hat{2}}|\Big\}\,, \tag{4.9}$$

with F^{++} a linear superfield (i.e. an analytic superfield under the same constraints that G^{++} obeys). This gives

$$\frac{1}{2}\int \mathrm{d}\zeta^{(-4)}\,(G^{++})^2 = \int \mathrm{d}^8 z\,\Big\{\bar{\Phi}\Phi - \frac{1}{2}\,G^2\Big\}\,. \tag{4.10}$$

More general kinetic terms for the tensor multiplet, eq. (3.24), are easier to analyze using the projective superspace techniques [25, 26]. In projective superspace, the $\mathcal{N}=2$ supersymmetric action involves integration over a closed loop in the complex plane $\mathbb{C}$, unlike the harmonic superspace action (3.24) that involves integration over S^2. The projective-superspace action for the $\mathcal{N}=2$ tensor multiplet [25, 26] is

$$S_{\mathrm{P}} = \frac{1}{2\pi\mathrm{i}}\oint_\gamma \frac{\mathrm{d}w}{w}\int \mathrm{d}^8 z\,\mathcal{L}(\Sigma(w), w) + \text{c.c.} \tag{4.11}$$

Here $\mathcal{L}$ is an arbitrary "good" function, γ an appropriately chosen contour, and

$$\Sigma(w) = \Phi + w\,G - w^2\,\bar{\Phi}\,. \tag{4.12}$$

The action (4.11) can be shown to be $\mathcal{N}=2$ supersymmetric, in spite of the fact that it involves integration only over the $\mathcal{N}=1$ superspace. The relationship between the harmonic action S_{H} in (3.24) and the projective action (4.11) was studied in [28].

As an example, let us consider the special choice [29] that corresponds to the so-called c-map [30]: $\mathcal{L}(\Sigma, w) = \Xi(\Sigma)/w^2$, with Ξ a holomorphic function. In addition, the contour γ in (4.11) should now enclose the origin. Then we obtain

$$\frac{1}{2\pi\mathrm{i}}\oint_\gamma \frac{\mathrm{d}w}{w}\int \mathrm{d}^8 z\,\frac{\Xi(\Sigma(w))}{w^2} = -\int \mathrm{d}^8 z\,\bar{\Phi}^I\,\Xi_I(\Phi) + \frac{1}{2}\int \mathrm{d}^8 z\,G^I G^J\,\Xi_{IJ}(\Phi)\,, \tag{4.13}$$

where we have specialized to the case of several tensor multiplets. Comparing the first terms on the right of (4.8) and (4.13), we see that they are of the same functional form. We therefore believe that $\mathcal{N}=2$ supersymmetric theories of the form

$$S = \frac{1}{2\pi\mathrm{i}}\oint_\gamma \frac{\mathrm{d}w}{w}\int \mathrm{d}^8 z\,\frac{\Xi(\Sigma(w))}{w^2} + \int \mathrm{d}^8 Z\,\Upsilon(\Psi, \mathbb{W}) + \text{c.c.} \tag{4.14}$$

that are generated by two holomorphic potentials, Ξ and Υ, deserve further study.

In the main body of this note, we studied models for a single massive tensor multiplet in $\mathcal{N}=1$ and $\mathcal{N}=2$ supersymmetry. The results can be clearly extended to the case of several mutiplets.

Shortly before the submission of this note to the hep-th archive, we received a new paper [31] in which the supersymmetric Freedman-Townsend models [7, 32] (see also [35]) and their generalizations [33] were made massive by extending the non-supersymmetric construction of [34].

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A. $\mathcal{N}=2$ superspace integration measures

Here we introduce various $\mathcal{N}=2$ superspace integration measures used throughout this paper. They are defined in terms of the spinor covariant derivatives D^i_α and $\bar{D}^{\dot\alpha}_i$, with $i=\hat{1},\hat{2}$,

$$\{D^i_\alpha, D^j_\beta\} = \{\bar{D}^{\dot\alpha}_i, \bar{D}^{\dot\beta}_j\} = 0\,, \qquad \{D^i_\alpha, \bar{D}_{j\dot\beta}\} = -2\mathrm{i}\,\delta^i_j\,(\sigma^m)_{\alpha\dot\beta}\,\partial_m\,, \tag{A.1}$$

and the related fourth-order operators

$$D^4 = \frac{1}{16}(D^{\hat{1}})^2(D^{\hat{2}})^2\,, \qquad \bar{D}^4 = \frac{1}{16}(\bar{D}_{\hat{1}})^2(\bar{D}_{\hat{2}})^2\,. \tag{A.2}$$

Integration over the chiral subspace is defined by

$$\int \mathrm{d}^8 Z\, L_{\mathrm{c}} = \int \mathrm{d}^4x\, D^4 L_{\mathrm{c}}\,, \qquad \bar{D}^i_{\dot\alpha} L_{\mathrm{c}} = 0\,. \tag{A.3}$$

Integration over the full superspace is defined by

$$\int \mathrm{d}^{12} Z\, L = \int \mathrm{d}^4x\, \bar{D}^4 D^4 L\,. \tag{A.4}$$

In terms of the harmonic-dependent spinor derivatives

$$D^\pm_\alpha = D^i_\alpha\, u^\pm_i\,, \qquad \bar{D}^\pm_{\dot\alpha} = \bar{D}^i_{\dot\alpha}\, u^\pm_i\,, \tag{A.5}$$

and the related fourth-order operators

$$(D^+)^4 = \frac{1}{16}(D^+)^2(\bar{D}^+)^2\,, \qquad (D^-)^4 = \frac{1}{16}(D^-)^2(\bar{D}^-)^2\,, \tag{A.6}$$

integration over the analytic subspace is defined by

$$\int \mathrm{d}\zeta^{(-4)}\, L^{(+4)} = \int \mathrm{d}^4x \int \mathrm{d}u\,(D^-)^4 L^{(+4)}\,, \qquad D^+_\alpha L^{(+4)} = \bar{D}^+_{\dot\alpha} L^{(+4)} = 0\,. \tag{A.7}$$

Integration over the group manifold SU(2) is defined according to [18]

$$\int \mathrm{d}u\, 1 = 1 \qquad \int \mathrm{d}u\, u^+_{(i_1}\cdots u^+_{i_n}\, u^-_{j_1}\cdots u^-_{j_m)} = 0\,, \quad n+m>0\,. \tag{A.8}$$

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